APPENDIX B

MEASURING ASSESSMENT UNIFORMITY FROM MARKET SURVEY DATA: WEIGHTED COEFFICIENT OF DISPERSION

1. Computing the Coefficient of Dispersion

The coefficients of dispersion (CODs) contained in this report are calculated from the estimates of market value (sales, appraisals, and Computer Assisted Mass Appraisal (CAMA) estimates) derived in the New York State Board of Real Property Services' 2009 market value survey. "Weighted" CODs for the entire assessment roll are calculated when appraisals and/or sales are available and are used to reflect the composition of each assessment roll appropriately, as rolls may be stratified by property type and value category for survey purposes. In contrast, CODs calculated through CAMA need not be weighted as they use data for all the parcels for which values are being predicted.

The general formula for a coefficient of dispersion around the median assessment ratio is:

$$(1) \quad COD = \frac{100}{R_m} \left(\frac{\sum_{1}^{N} \left| R_i - R_m \right|}{N} \right)$$

where:

COD = coefficient of dispersion, i.e., the average percent of dispersion around the median assessment ratio;

R_m = median assessment ratio;

R_i = observed assessment ratio for each parcel;

N = number of properties sampled.

This general formula is usually applied in estimating the COD from non-stratified sales data, where the true representativeness of each sale is unknown. Where a sales ratio analysis was used directly in the survey (residential property only), the formula above describes the residential COD calculation accurately. Where a stratified sample was used and the

representativeness of each sampled parcel is known, the formula can be modified by weighting each of the observed assessment ratios as follows:

Let i = the sampled parcel, j = the stratum, and R_{ij} = the assessment ratio of the ith parcel in the jth stratum.

Let w_j = p_j / s_j, where:

p_j = the total number of parcels on the assessment roll in the jth stratum;

s_j = the number of sampled j^{th} stratum.

Let $\overline{w}$ = the total number of parcels on the roll divided by the total number sampled (i.e., the reciprocal of the overall sampling ratio).

The weight (w_j) is calculated for each stratum, and is identical for all sampled parcels within it. For example, in a municipality where there are 600 residential parcels in the assessed value range of $40,000 to $80,000, and six of them are selected in a random sample, then each of the six sample ratios would have a weight of 100 because it is assumed to represent 100 of the parcels in that range (stratum).

Since i signifies the sampled parcel and j the stratum it was selected from, the assessment ratio for a given observation will thus be Rij. As in the case of formula (1) above, we must calculate the absolute difference between Rij and Rm. Then, these differences are adjusted to reflect the composition of the entire roll rather than the sample by applying to each the ratio $wj/\overline{w}$. For all observations within each of the i strata, the formula for the weighted coefficient of dispersion around the median thus becomes:

$$(2)\quad COD_w = \frac{100}{R_m}\left(\frac{\sum_{1}^{i}\sum_{1}^{j}\frac{w_j}{\overline{w}}\left|R_{ij} - R_m\right|}{N}\right)$$

The procedure for calculating the weighted coefficient for each assessing unit entails the following steps.

1. Calculate the assessment ratio (R_{ij}) for each sample parcel by dividing the assessed value by the estimated market value.

2. Array the assessment ratios from lowest to highest within each assessing unit.

3. Calculate the weight (w_i) for each stratum and $\overline{w}$, representing the total number of parcels on the roll divided by the size of the sample.

4. Normalize the weight of each sampled parcel by dividing by ($\overline{w}$).

5. Select the median assessment ratio (R_m) from the weighted list (length of list equals the total number of parcels sampled.)

6. Apply equation (2) above.

It is important to note that the median assessment ratio as used in equation (2) will not necessarily be the same as the median of the sampled ratios, i.e., as used in equation (1). The former median, from the "weighted" list of appraisals, reflects the weighting applied to achieve equal representativeness in the population.

For cases where the stratification process is embedded even further, such as multiple school district portions within an assessing unit, the calculations embodied in these equations entail additional subscripts. However, the general form of the equation remains the same. Once again, the purpose of weighting is to correct, to some degree, the deficiencies of the sampling procedures from the standpoint of measuring uniformity, i.e., to construct a measure built upon equally-likely selection of each parcel from an assessment roll.

In instances where CAMA model estimates were used in lieu of regular appraisals (residential property only), the COD calculation procedure was modified as follows: (1) a residential COD was computed for the modeled residential parcels, according to the formula in equation (1) above; (2) a non-residential COD was computed for the remaining parcels using the formula given in equation (2) above; and (3) to compute an all-property weighted COD, these two COD estimates were combined through weighting them according to relative parcel numbers.

II. Computing the Price-Related Differential

The price-related differential (PRD) is used to determine if there is a price-related bias in a municipality's assessing practices. It compares the simple mean of the assessment ratios to the price-weighted mean ratio. If no bias exists, the two figures will be virtually equal and the PRD would be close to one (1.0), indicating assessment neutrality. If a municipality tends to over-assess higher valued properties, the price-weighted mean will be higher than the simple mean and an index of less than 1.0 will result (indicating progressivity). The opposite occurs

when higher-valued properties are consistently under-assessed. In this case, the price-weighted mean will be lower than the simple mean and the result will be an index which is greater than 1.0, indicating regressivity.

Example of Price-Related Differential Values

	Regressive **Greater than 1.03**	**Neutral** **0.98 to 1.03**	**Progressive** **Less than 0.98**
Ratios:			
a. Simple Mean	.70	.70	.70
b. Price-weighted Mean	.58	.68	.85
Price-Related Differential (a / b)	1.21	1.03	0.82

The formula for the price-related differential uses the same weighting process previously described in relation to the coefficient of dispersion:

$$(3)\quad IR = \frac{\dfrac{\sum_{1}^{i}\sum_{1}^{j}\left(R_{ij}\bullet\dfrac{w_j}{\overline{w}}\right)}{N}}{\dfrac{\sum_{1}^{i}\sum_{1}^{j}\left(ASV_{ij}\bullet\dfrac{w_j}{\overline{w}}\right)}{\sum_{1}^{i}\sum_{1}^{j}\left(APV_{ij}\bullet\dfrac{w_j}{\overline{w}}\right)}} = \frac{\text{Simple Mean}}{\text{Weighted Mean}}$$

where:

N = the total number of sampled properties;

i = the sampled parcel;

j = the stratum;

w_j = the weight of every sampled parcel drawn from the jth stratum (see previous discussion of sample weighting in relation to COD);

$\overline{w}$ = the total number of parcels in a stratum divided by the total number sampled in that stratum (see previous discussions of sample weighting in relation to COD;

R_{ij} = ratio of assessed value to estimated market value (appraisal or sale) (one for each sampled property in each stratum);

ASV_{ij} = assessed value of the “i^{th}” sampled property in the jth stratum; and

EMV_{ij} = estimated market value of the "i^{th}" sampled property in the jth stratum.